Section 11.2 Arithmetic Sequences and Series - Discovery

Now that we know what arithemetic sequences and series are, time to answer some important questions regarding them. First up, when do they converge?

1. Convergence of Arithmetic Sequences

   (a) If you take zero, and add it to itself infinitely many times, what would happen?

   (b) If you take any real d number besides zero, and add it to itself infinitely many times, what would happen?

   (c) Keeping the above answers in mind. When would an arithmetic **sequence** with common difference d converge and when would it diverge?

2. Convergence of Arthmetic Series

   (a) Now consider an arithemetic **series**. If you add all the terms in an arithmetic sequence together, when would it be finite and when would it be infinite?

   (b) So, when does an arithmetic series with common difference d converge, and when does it diverge?

Now, as we have just seen, infinite arithmetic series are pretty boring in terms of when they converge or diverge. However, if we look at a finite arithmetic series, things become more interesting.

One famous story in mathematics concerns Carl Friedrich Gauss ($1777 - 1855$; the man partly responsibible for your favorite method of solving linear systems). When he was 10 years old, his teacher gave the pupils in an arithmetic class the problem of summing the integers from 1 to 100. Gauss came up with the answer almost immediately: 5050. To obatin this answer he added the first number to the last to get $1 + 100 = 101$, then he added the second number to the next to last to get $2 + 99 = 101$. He saw that he would continue to get a sum of 101 until I formed the last sum of $50 + 51 = 101$. This would give 50 sums of 101 for a total of $50(101) = 5050$.

What Gauss did was to discover a quick means of summing a finite geometric series. He discovered that $1 + 2 + 3 + \cdots + n = n(n+1)/2$, a formula that we have already proven using triangular numbers.

1. Some accounts of the story state that Gauss's teacher was not amused and then gave Gauss (and the class) more challenging finite arithmetic series to sum such as

$$\sum_{i=1}^{100} 3i + 100 = 103 + 106 + 109 + \cdots + 400$$

   Gauss was able to solve each of these immediately, while his classmates could not even get the correct answer. Are you clever enough to find a means of evaluating the above sum? Even if you said 'no' to avoid doing work, try it!

2. Test your solution method to solve each of the following:

   (a)
$$\sum_{i=1}^{50} 5i + 8 = 13 + 18 + 23 + 28 + \cdots + 258$$

   (b)
$$\sum_{i=20}^{100} 4i + 1 = 81 + 85 + 89 + 93 + \cdots + 401$$